[10] Blind radial distortion compensation in a single image using fast Hough transform

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Abstract

In this paper, we present an automatic technique for compensation of radial distortion, which is characteristic of wide-angle lenses. The proposed method estimates distortion parameters using a single image from unknown source. No calibration objects are required, but it is assumed that the original scene contains straight lines. The method is based on finding such radial distortion parameters, that maximize total length of linear segments. We employ a fast Hough transform to estimate the overall curvature of lines without selecting any. The proposed algorithm is tested on real images obtained using calibrated camera lenses with different radial distortion. For the formal evaluation of the algorithm, we propose a quality measure for geometric distortion compensation, which works correctly even in the case when the problem of determining the coefficients is ill-conditioned.

Keywords: digital image processing, image analysis, lens system design, radial distortion, automatic calibration, fast Hough transform.

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Introduction

Smart processing of images and video streams has found its way to a wider range of applications in many industries. Meanwhile optimal practical application includes using photo and video cameras with low quality lenses that feature significant distortions, and such distortions are to be compensated in the image post-processing stage. Such compensation requires distortion parameters information. In some cases it can be found in the camera specifications, but most likely optical system needs to be calibrated to determine such parameters.

This paper will examine the problem of compensation of radial distortion – an optic system imperfection caused by the spherical shape of the camera lens. Radial distortion breaks the geometric similarity between an object and its image, straight lines on the image become curved (except for the lines crossing the frame optical center), and the degree of curvature increases from the center to the periphery. Images obtained by the wide-lens cameras not designed for measurement purposes are primarily subjected to radial distortion.

The most common camera distortion compensation method is pre-calibration when a special calibration object is located in the camera coverage area [1, 2]. Calibration objects can include periodic structures [1] and random texture [2] with specific statistical properties [2]. Characteristics of such object must be known in advance.

Methods that do not require a special calibration object are also applied, but they use several recorded images of the same scene. Such methods are based on the priori information about camera moves or the scene geometry and achieve calibration factoring in the epipolar geometry limitations [3, 4].

But in many real life situations such methods are not applicable since the required images cannot be obtained from the camera. This happens when only a scene image is available but the information on the camera used is not. In such case distortion parameters may be estimated based on the image analysis. Such analytical method of correcting the geometric distortions is called automatic (or blind) calibration. These methods include image analysis in the frequency domain [5], image structure research [6] and others. But the most common methods are based on the assumption that a typical scene included a large number of straight lines that remain straight at central projection but become curved due to radial distortion. Such assumption is characteristic for many practical tasks, e.g. visual navigation of aircraft [7-9]. Papers [10-12] propose to perform test compensations of radial distortion with various parameter assumptions and evaluate the line curvature at the reconstructed images via Hough image calculation. Paper [13] proposes similar approach but histogram of oriented gradients is calculated instead of the Hough image. Authors of [14] do not perform such reconstruction but detect the lines and approximate extracted curves with circle arcs and subsequently calculate the radial distortion parameters.

This work presents further development of algorithm [12] based on fast Hough transform (FHT) [15, 16].

1. Radial Distortion Compensation Algorithm

Like its predecessor the proposed method performs test compensations of radial distortions within various distortion parameters assumptions and then uses the FHT to evaluate the corrected image quality. Factoring the curve line element changes into radial distortion correction ensured higher algorithm accuracy and permitted to shift to three-parameter model of radial distortion.

1.1. Distortion Model

Assume that the lens system optical axis crosses the image in its geometric center. Locate the origin of coordinates in the center and direct x and y axes to the right and down accordingly. Use the classic Brown modes [17] as the distortion mathematical model. In this model the coordinates on the radially distorted image are expressed via known coordinates on the original (undistorted) image as follows:

$$\begin{cases} r_d = f(r) = (1+\sum_{i=1}^{n} k_i r^{2i})r \\ x_d = x \cdot r_d / r \\ y_d = y \cdot r_d / r \end{cases}, \quad (1)$$

where $r=\sqrt{x^2+y^2}$, $k_i = 1...n$ –distortion parameters, (x, y) – original location of the dot, (x_d, y_d) – dot location resulted from the radial distortion.

If $r_d < r$, such effect is called barrel distortion, otherwise (when $r_d > r$) – pincushion distortion. Thereat barrel distortion is common for wide-angle lenses. Pincushion distortion of long-focus lenses is less common for non-professional environment (where lens parameters data can be lost) and will not be covered in this work.

Value of factors k_i depends on the camera focus length but this dependence will not be discussed in this paper since focal length changes would not change the radial distortion itself but rather its parameterization.

When the i serial number grows the expansion coefficients k_i of the real optic systems are falling steeply, so we can review only a few first expansion terms. We shall limit the review of radial distortion parameters up to $n = 3$.

1.2. Distortion Compensation at the Known Parameters

Given a radially distorted image with the known distortion parameters. Let us review the task of generating an image with compensated radial distortion. Let us use formula (1) to transform the coordinates (x, y) of each pixel of the generated image I. Brightness value in the pixel with coordinates (x_d, y_d) on the distorted image I_d will be the target value for the pixel on the generated image. Since image I_d is defined only in the integer-valued coordinate grid nodes, the integer values for non-integer x_d and y_d must be obtained by interpolation of function I_d.

Note that the image conversion reverse to radial distortion required only direct coordinate transform (1) without transform inversion.

1.3. Method of Evaluating Radial Inversion Parameters

As mentioned above the reconstruction algorithm will be based on evaluating the quality of distortion correction at all possible values of distortion parameters at some grid. Now let us describe the quality evaluation method at the known test parameters of distortion. For these ends we will need to set forward some specific details of using the fast Hough transform technique.

1.3.1. Method Idea

Hough transform consists of summing the image pixel values along various straight lines crossing the image. On several occasions Hough image calculation has been suggested for evaluation of image lines straightness [10, 11], but was not applied due to seeming computational complexity of the most common algorithm [18, 19]. This paper applies the same approach but suggests to use fast Hough transform.

Assume that the original image size is $n \times m$. For fast Hough transform predominantly mostly vertical straight lines (with deviation angle tangent limited by one) of an original image are parametrized by the point of intersection with the image top edge s and point of intersection with the image bottom edge $s+t$. Let us call value t, which characterizes the straight line deviation angle, a tangenta, $t \in [-n, n]$. Each pertinently vertical line in the Hough space has a corresponding point with coordinates (s, t). Let us determine matrix $H^{ver}_{s,t}$ of $m \times (2n-1)$ size, where the value of each element is equal to the sum of values of the pixels of the original image along the corresponding straight line. Similar parametrization is used for predominantly mostly horizontal straight lines, the only difference being that the left and right image edges are used. The size of resulting matrix of sums $H^{hor}_{s,t}$ is $n \times (2m-1)$. A pair of matrixes (H^{ver}, H^{hor}) forms a whole Hough image H.

Assume that the original image includes (possibly non-continuous) bright long section on the straight line parametrized as (s_0, t_0), and that there are no similarly bright and long sections in its vicinity. In this case the value of brightness sum along the line (s_0, t_0) will be large and this value along the adjacent lines will be small. Therefore the Hough image will record a sharp peak in the point s_0 on line $t = t_0$. Assume now that the original image includes a bright curved section. Since there are many straight lines of similar parameters which are tangent to the curved section, the sums along these lines will have close values and a sweeping "hill" will be visible on the Hough image.

Since the real life scenes usually contain straight lines and there is a slim chance that radial distortion reconstruction at arbitrary parameters would randomly reconstruct some curved line to a straight line, then the best radial distortion parameters of an image shall be such parameters when the lines of reconstructed image are straighter, i.e. its Hough image contains sharp peaks rather than blurred "hills".

Now we shall describe a specific algorithm for calculating the functional of the distortion correction quality assessment based on fast Hough transform.

1.3.2. Correction of Distortion while Maintaining Intensity Integral

Assume there is a scene image containing straight lines subjected to radial distortion and there are distortion parameters. Let us set a problem of evaluating whether the image is radially distorted at these particular parameters. For these ends select the image boundaries by calculating the brightness gradient modulus in each dot and then perform trial image reconstruction as described in subparagraph 1.2. Now perform fast Hough transform of the reconstructed image. As mentioned above in case these radial distortion parameters are correct, the bright long sections of straight lines on the reconstructed image would show sharp peaks on the Hough image; the longer and brighter the line sections are, the more visible this peak is against the background of flat "hills" that correspond to curved boundaries. But note that the radial distortion transformation does not maintain length and area, so the integral of the section intensity may randomly change depending on its location on the image. Moreover, if the radial distortion correction turned a curve into a straight line while maintain the same distance between the section ends, its length will be reduced which would cause reduction of the integral intensity and the amplitude drop of the corresponding peak in the Hough space, and would also result in indirect "penalizing" of the correct parameters. Therefore to improve the accuracy of the line straightness based on the corresponding Hough space peaks it is reasonable to develop such method of radially distorted image reconstruction that would maintain integral image boundary intensity when geometry is reconstructed.

Let us assume there is the distorted image I_d. Create blank (where all values are equal to zero) image I. Assume there is coordinate transformation reverse to (1), which allows finding coordinates (x, y) on image I' for each pixel (x_d, y_d) of the distorted image. Then perform the following operation for each pixel of the distorted image I_d

$$I'(x, y) = I'(x, y) + I_d(x_d, y_d).$$

The proposed image transformation satisfies the stated requirement of maintaining the integral intensity along the curves, although it is not applicable for visualization since it does not maintain the point value. For this reason it is not used in this work for trial image reconstruction, but rather a standard transformation described in subparagraph 1.2 is used to create the output image based on the identified distortion parameters.

As mentioned above the trial reconstruction that maintains integral intensity along the curved lines requires calculation of the coordinate transformation reverse to (1). But in case (1) is not monotone the reverse transformation does not exist. For this reason it is reasonable to limit possible values of the distortion parameters k_i in such a way as to make transformation (1) monotone at the section $[0, r^*]$, where r^* is such radius when $f(r^*)$ is not less than certain predetermined value r_{crit}. The circumference of this radius would be called critical.

If a circle of r_{ext} radius circumscribed around an image is accepted as critical, the distortion parameters limitations are too strict to the effect that heavily distorted images from the test set are not described with reasonable accuracy by the polynomial model with small number of elements. If the critical circumference radius is too small (e.g. it is equal to the radius r_{int} of the circle escribed into the circle image) the obtained distortion correction parameters estimate will be meaningless since the distortions of the significant image peripheral areas may be arbitrarily large. In our work the critical circle radius of $r_{orit} = 0.7 \times r_{ext}$. is selected. Meanwhile the image aspect ratio is 3:4, which yields $r_{int} = 0.6 \times r_{ext}$. For other aspect ratios the reasonable critical circle radius selection will probably be the value that divides the section $[r_{int}, r_{ext}]$ in similar proportion.

Note that the distortion parameters impact the global image scale, so part of the scene may extend beyond the borders of the reconstructed image, which would infract the frame integral intensity. Let us introduce the notion of fixed circumference. It will designate the non-degenerate circle with the radius not changed by trial correction of radial distortion. At some distortion parameters such circle may be non-existent. Now at the fixed distortion parameters let us introduce additional scaling at the factor of k_0. For each circle not larger than critical at these distortion parameters the k_0 coefficient may be selected in such a way that the selected circle will be fixed (for scaling transformation).

Note that the monotoneness of transform $f(k_0 r)$ up to r_{crit} would not guarantee preservation of the image integral intensity within the critical circle if the latter extends beyond the image limits. In order to maintain the integral inside the truncated circle the additional requirement of $f(k_0 r) \le k_0 r$ shall be in force for section $[r_{int}, r_{crit}]$. Now to ensure integral intensity across the whole image it would be sufficient to clear to zero the gradient modulus values outside of the critical circle.

1.3.3. Calculating Reverse Coordinate Transform for Radial Distortion

The procedure proposed for test reconstruction of radially distorted image requires calculation of the coordinates transformation reverse to (1). Assume that the coordinates (x_d, y_d) of the distorted image pixel and radial distortion parameters are predetermined.

Let us assume $r_d = \sqrt{x_d^2 + y_d^2}$ and rewrite equation (1) in the following form:

$$r_d = f'(r) = (1 + \sum_{i=1}^{n} k_i' r^{2i}) r \,. \tag{2}$$

Find scaling transformation with coefficient k_0 that preserves the critical circle radius:

$$k_0 f'^{-1}(r_{crit}) = r_{crit} \,. \tag{3}$$

Taking into account scaling at k_0 and $k_i = k_0 k'_i$ replacement for $i > 0$ the coordinates transformation will appear as:

$$r_d = f(r) = (\sum_{i=0}^{n} k_i r^{2i}) r \,. \tag{4}$$

There is no analytical solution for equation (4) regarding r at $n > 1$, therefore computational solution must be sought. Since reverse transformation at fixed distortion parameters needs to be computed repeatedly (complexity is directly proportional to the image area), it would be reasonable to create a crude model of reverse transformation and use it instead of computational solution of equation (2) in each point. Since the radial distortion transformation for the lenses under examination turned out to be sufficiently smooth, and the points are symmetrically offset against the radial distortion center, the value of function $r = f^{-1}(r_d)$ was numerically evaluated for m of equidistant values of r_d at $[0, r_{crit}]$ section and linear interpolation was used to compute r for all dots at the input image. m was equal to 300 in the numerical experiments shown below.

1.3.4. Angular Image Descriptor

As mentioned above straight lines of the image generate sharp peaks on the Hough image, and curved lines generate more blurred peaks. In order to remove blurred peaks and preserve sharper peaks let us deduct from Hough image the same image but smoothed out with Gaussian filter along the t axis thus preserving only non-negative values:

$$\begin{aligned} P^{ver} &= \max(0, H^{ver} - G_t(\sigma) * H^{ver}), \\ P^{hor} &= \max(0, H^{hor} - G_t(\sigma) * H^{hor}). \end{aligned} \tag{5}$$

where $G_t(\sigma)$ – Gaussian filter kernel with root-mean-square deviation σ. In the experiments below σ was assumed as equal to 5.0 with the image width of 360 pixels.

Now define vectors F^{ver} of $2n-1$ length and F^{hor} of $2m-1$ length so that each of their elements would contain straightness estimate for the set of edges of the certain tangenta t. For such estimate let us take value dispersion in line t of filtered Hough images P_t^{ver} and P_t^{hor}.

Dispersion is used to evaluate the straightness because blurred "hills" produce lesser dispersion as compared to sharp peaks. The dispersion is also growing when the number of peaks increases.

Now take into account the fact that the straight lines passing through the image center are not affected by any radial distortion and the lines passing closer to the center are less distorted than the far lines. As far as discretization and other interferences create noise on the Hough image peak amplitude it would be reasonable to assign less weight to the peaks that correspond to the straight lines located closer to the image center.

Based on the aforesaid the straightness estimate for a beam of parallel lines of each tangenta t would appear as follows:

$$F_t^{ver} = \sigma^2 (P_{s,t}^{ver} W_{s,t}^{ver}),$$
$$F_t^{hor} = \sigma^2 (P_{s,t}^{hor} W_{s,t}^{hor}), \quad (6)$$

$$\sigma^2 (P,W) = \frac{\sum_s P_{s,t}^2 W_{s,t}}{\sum_s W_{s,t}} - \left(\frac{\sum_s P_{s,t} W_{s,t}}{\sum_s W_{s,t}} \right)^2, \quad (7)$$

where W^{ver} and W^{hor} – distance from the straight line (s, t) to the image optical center. Let us use the term *angle descriptor* to describe this reduction of Hough images H^{ver} and H^{hor} to vector $F = F^{ver} \cup F^{hor}$.

1.3.5. Distortion Correction Evaluation Based on the Angular Descriptor

We expect to see peaks from the quasi-parallel edge beams on the angular descriptor, besides, their straightness affects both peaks amplitude and fall velocity. Let us use the signal entropy value as the peak strength measure that takes into account both of these effects.

Assume F has k levels of values and frequency of level k appearance is $P(F_k)$. Then the F signal entropy is defined as follows:

$$E(F) = -\sum_{k=1}^{K} P(F_k) \log P(F_k). \quad (8)$$

This equation was used in this work to evaluate the distortion correction quality – the less is E entropy value, the stronger are the peaks in the angular descriptor. This is expected to be related to the fact that the straight lines on the image are less distorted.

1.4. General Description of Blind Compensation Algorithm

The proposed algorithm uses the n-dimensional space exhaustive search of the radial distortion parameters (experiments conducted at n=3) within the predetermined limits. The experiment included the following steps:

1. Generate an outline image selecting the input image boundaries by calculating the brightness gradient modulus in each point of the image.
2. Set to zero outline image pixels outside of the critical circumference (see p. 1.3.2).
3. Perform the search in the parameters space within the predetermined limits. For each parameters set:

1.1. Check the distortion transformation limitations described in p. 1.3.2 at this set of parameters; in case limitations are not satisfied proceed to the next set of parameters.

1.2. Calculate reverse coordinate transformation (p. 1.3.3).

1.3. Complete reverse distortion transformation of the outline image while preserving the intensity integral (p. 1.3.2)

1.4. Generate angular descriptor (p. 1.3.4).

1.5. Evaluate the distortion correction based on the angular descriptor (p. 1.3.5).

4. Find the parameters set that minimizes the target function value (8).
5. Use the identified parameters to obtain output image by compensating the radial distortion of the input image (p. 1.2).

2. Experimental Verification of the Proposed Algorithm Quality

Algorithm was tested on the document images and three-dimensional indoors and outdoors scenes containing straight lines (Figures 2 *a*-*d*). Cameras with 6 different lenses were used to photograph the images (Table 1). For control purposes the algorithm was applied to the images described in [12]. Using MATLAB Single Camera Calibration software module with Zhang [1] algorithm reference radial distortion parameters were obtained for each data set, except for the data taken from [12]. A fixed transparent overlay with 1 meter side and 10 cm pitch staggered pattern.

The experiment objective was quantitative evaluation of the distortion parameters definition accuracy for various modifications of the proposed algorithm on the multitude of test images. Note that using simple metrics in the distortion model parameters space is not the best option for quantitative evaluation. Firstly there is no apparent method to collate the weight of equal deviations for different model coefficients. Secondly, and more importantly, two polynomials with significantly different sets of coefficients may have only small difference at the section under our examination. For practical reasons the error of any distortion compensation algorithm is directly related to points coordinates deviations after correcting relatively perfect coordinates in the image area. Quantitative method of residual distortion strength evaluation will be described below, as well as the results of the algorithm based on such method.

2.1. Method of Evaluating the Distortion Strength and Compensation Correctness

Assume that we know the initial point position $\vec{r} = (x, y)$ before distortion and the observed point position $\vec{r}_d = (x_d, y_d)$ impacted by certain distortion. The coordinates of reconstructed point location are $(x', y') = f^{-1}(\vec{r}_d)$, where f^{-1} – the estimated function of distortion compensation. Let us consider the Euclidean norm of reconstructed coordinates deviation in the perfect image coordinates:

$$\tilde{\Delta}_f(x, y) = \| \vec{r} - f^{-1}(\vec{r}_d) \|_2 \tag{9}$$

We introduce regular rectangular grid N×M (Figure 1*a*) and define the distortion compensation quality evaluation as an average deviation norm in the grid nodes:

$$\tilde{d}_f = \frac{1}{NM} \sum_{i,j=1}^{N,M} \tilde{\Delta}_f(x_i, y_j). \tag{10}$$

As mentioned above the scale notion is hardly definable for the distorted images. Besides, the image reconstructed from the distorted rectangular image (Figure 1 *b*) would not be rectangular (Figure 1 *d*). In practice this requires selection of the scale (and generally shift) of the output image rectangle, optimal in terms of specific application. In this respect overall reconstructed image extension or compression shall not affect reconstruction quality assessment. For this reason the quality assessment is modified as follows:

$$\Delta_{f,p}(x, y) = \| \vec{r} - L_p(f^{-1}(\vec{r}_d)) \|_2$$
$$d_f = \min_p \frac{1}{NM} \sum_{i,j=1}^{N,M} \Delta_{f,p}(x_i, y_j), \tag{11}$$

where L_p – scaling and shifting transformation (in our case – only scaling) with parameters *p*.

Similarly we define the strength degree of the initial image distortion d_0 as the assessment of reconstruction quality without any compensation (Figure 1c):

$$d_0 = d_f \,|_{f(\vec{r}) = \vec{r}} \,. \tag{12}$$

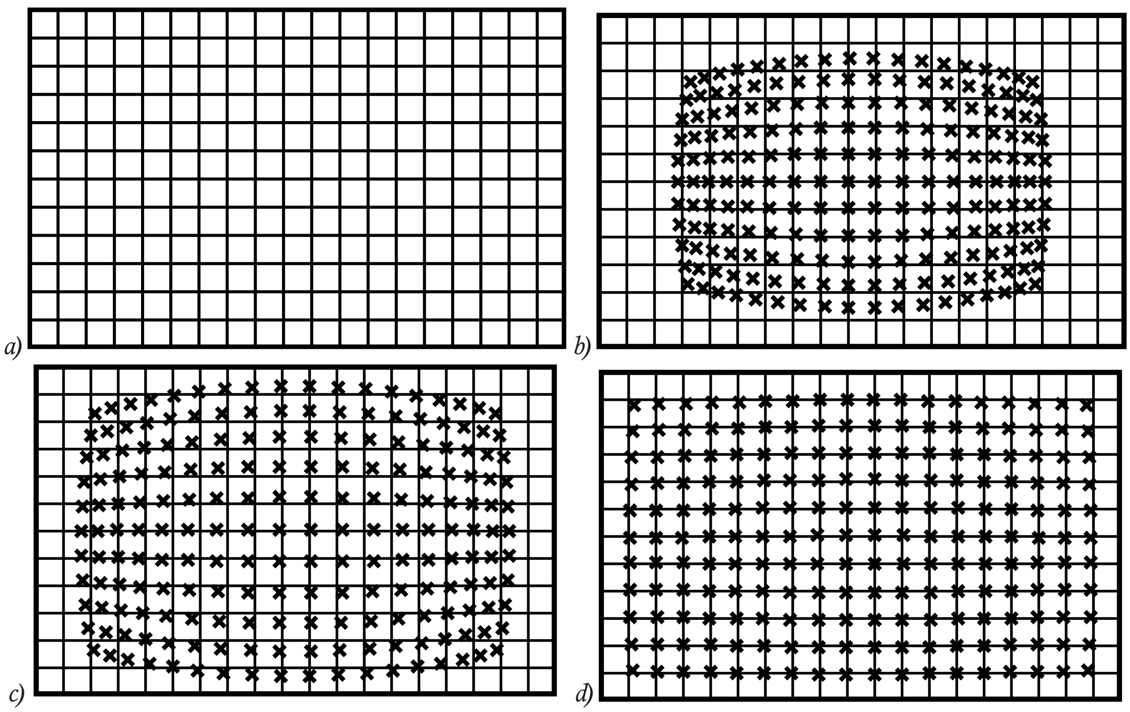

Fig. 1. Evaluation of the radial distortion compensation: a) grid used, b) radial distortion results, c) transformation with the same distortion parameters but with optimal scaling used for computing; d) distortion compensation with optimal scaling used to compute d_f

Now we can introduce relative evaluation of the distortion compensation quality:

$$Q_f = 10 \cdot [1 - (d_f / (d_0 + \varepsilon))], \tag{13}$$

where 10 – coefficient used to reduce the evaluation to the ten point scale, and ε – parameter that governs the tolerance of the low distortion reconstruction evaluation. Its value may vary depending on the reconstruction accuracy requirements. Thus the introduced relative assessment permits to obtain comparable results at the image sets with significantly different distortion levels.

In this work uniform grid of 36×48 nodes and $\varepsilon = 1.0$ with the image size of 360×480 were used to calculate (13).

2.2. Discussion of Experimental Results

The set of experiments were conducted to compare results of different variants of the proposed algorithm. The reproduced results of the algorithm shown in [12] were used as a reference. Each successive algorithm modification was cumulatively applied to the previous version. In the first experiment the algorithm proposed in [12] was upgraded by replacing the radial distortion model with the two-parameters model, and replacing the test reconstruction radial distortion conversion method with the method described in p. 1.3.2 except for scaling that preserves the critical circumference radius. Such scaling was added in the second experiment. The third experiment replaced the distortion correction evaluation functional described in [12] with the one proposed in p. 1.3.5 except for assigning different weights to peaks on the Hough image depending on the line distance from the image center (p. 1.3.4). Such weight assignment was added in the fourth experiment. The fifth experiment replaced the radial distortion model with the three-parameters model.

Table 1 shows the specifications of the optical systems used to obtain test data. Experiment results are presented in Table 2 and on Figure 2*e-h*.

Table 1. Characteristics of the data set used

Lens number	Number of shots	Distortion strength d_0
1	10	1.25
2	10	7.12
3	10	3.61
4	10	6.99
5	10	5.15
6	15	6,47

Conclusion

This work proposes the algorithm of reconstructing a radially distorted image based solely on the analysis of this image when a priori information about the camera parameters is not required. The algorithm was verified on real images obtained from the lenses of various radial distortion strength and proved to be practically applicable. The radial distortion correction accuracy significantly exceeded the accuracy of preceding algorithm proposed in [12] due, among other things, to using distortion model with larger number of parameters.

The proposed algorithm is based on minimization of the straightness estimation functional of the edges constructed over the result of the Hough transform of the trial distortion correction image for the gradient of the input image. As shown in [12] functionals of such type may be nonconvex therefore the optimization problem is resolved via full search in the radial distortion parameters space. Since radial distortion correction based on a single image from an unknown source is not the problem to be solved for big data flows it does not require high speed response so brute force search or exhaustive search is justified.

Table 2. Results of radial distortion compensation with different algorithm modifications (10-point scale, see p. 3.1)

№	Algorithm version	Average quality for the data set used: 1	2	3	4	5	6	Average quality for the version
0	Reference: algorithm proposed in [12]	8.4	2.6	8.6	0.8	4.6	2.0	4.50
1	The test reconstruction method was replaced with the one described in p. 1.3.2 without scaling; distortion model is extended to include two parameters	8.0	5.4	7.9	2.3	1.6	1.9	4.52
2	Scaling added	3.0	6.0	3.0	6.3	2.7	5.75	4.46
3	Distortion correction evaluation functional is replaced with the one proposed in p. 1.3.5 without weighing the peaks in the Hough image	6.3	6.2	7.8	6.5	7.1	4.0	6.32
4	Weighing added	9.0	7.8	9.0	7.8	7.9	7.75	8.21
5	Distortion model is extended to include three parameters	9.4	8.2	9.6	8.5	8.5	6.5	8.45

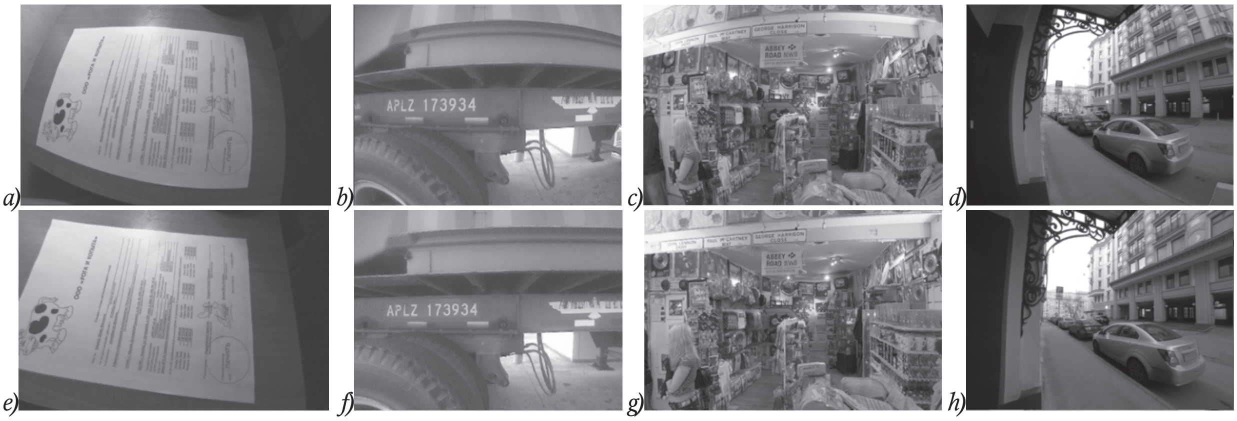

Fig. 2. Example of the algorithm operation: a–d) input images, e–h) images with corrected radial distortion generated by the final version of the developed algorithm

This work also contains a detailed discussion of key algorithm steps and includes the quality evaluation of different algorithm versions based on the proposed criteria. Quantitative and visual data were presented to illustrate that the proposed method can successfully handle both scene variability and different shooting conditions and distortion levels.

Further plans include acceleration of radial distortion parameters search process and adapting the proposed algorithm for video processing when the information from different frames is used for simultaneous optimization of radial distortion parameters.

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